Dynamics Of Double Pipe Heat Exchangers: Explicit Time Domain Solutions

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The dynamics of double pipe heat exchangers are governed by systems of nonhomogeneous hyperbolic partial differential equations when longitudinal dispersion effects are neglected and finite fluid capacitances accounted for. Their non-linear behaviour is described with a theoretical Hammerstein model with delays. The solutions are obtained in original variables by the characteristic, Laplace transform and difference equation methods (CLD) without numerical quadratures neither convolutions and valid for any dependence of heat transfer coefficients and capacitances from input variables. These solutions are also valid for generic non-zero initial conditions and any combination of stepwise variations of inputs, namely temperatures and flow rates of both fluids. The calculations are carried out with the aid of a double grid framework on the physical domain in order to allow for an arbitrary selection of sampling time and spatial coordinates. The results of the calculations are compared with those obtained by the finite element method (FEM) and by the numerical inversion of the Laplace domain solutions. The solutions compare very well with rigorous solutions.

1. Introduction

The dynamics of heat exchangers have been the objective of early studies on transient behaviour of industrial equipment (Klinkenberg, 1954; Mozley, 1956; Koppel, 1962; Ray, 1966). Studies have been continued on improving previous results (Tan and Spinner, 1978; Steiner, 1987) and emphasising new achievements (Yin and Jensen, 2003), mainly in rigorous and approximate analytical models (Tan and Spinner, 1991).
Though more difficult, solutions have been found in the case of countercurrent arrangement (Lachi, et al. 1997; Malinowsky and Bielsky, 2004) and on the hypothesis of flow maldistribution (Xuan and Roetzel, 1993).
The scientific literature report also semi-analytical methods (Tan and Spinner, 1984) and several numerical methods. The latter are based on the Finite Difference Method and the Method of Characteristics. The semi-analytical method (Tan and Spinner, 1984) is also based on the method of characteristics but the solution is obtained without iterations. Other studies, purposely developed for countercurrent mass transfer operations (Hwang, 1987) can also be valid for this case thank to the equivalent or equal, in dimensionless variables, resulting system of partial differential equations and boundary conditions (Tan and Spinner, 1984). More studies concern with changes in fluids temperatures, less studies with flow rate variations.
The present study is concerned in extending previous results and finding solutions valid for generic initial conditions and arbitrary combination of inputs without the cumbersome application of numerical quadratures neither convolutions. The objectives are reached with the combined application of the characteristic, Laplace transform and difference

equation methods (Evangelista, 2005). An explicit marching solution is obtained at each spatial location by antitransforming with the aid of difference equations and equivalent pulse transfer functions with zero-order hold element (Ogunnaike and Ray, 1994). One numerical solutions has also been developed and implemented for validation purposes, applying the finite element methods, FEM (Ames,1977) to time dependent problems. The resulting systems of ordinary differential equations are integrated with an implicit initial value integrator (Shampine and Reichelt, 1997). Another solution employed in the validation procedure has been developed. This solution is analytical in the Laplace domain and valid for any combination of step variations of inputs. However the solutions have been found numerically with an algorithm developed by Hollenbeck (1998).

2. Theory

Basic equations that govern the dynamic behaviour of this apparatus can be found by making a heat balance on a differential element *dz* as shown in Fig. 1a. In order to simplify the derivation the following assumptions are made:

1) The flow of both streams is highly turbulent for the resistance to the transfer of heat to be concentrated in thin boundary layers next to the wall ;
2) Physical, transport properties and heat transfer coefficients may vary arbitrarily with input variables;
3) Limited temperature differences so physical, transport properties and transfer coefficient can be considered constant with the axial dimension;
4) Wall resistance and capacitance negligible;
5) Axial temperature gradients much lower than the radial gradients in the boundary layers, so the diffusive axial transport be neglected in comparison to the convective transport;
6) Cross section and heat transfer areas are constant;
7) Entry length negligible in comparison to the length of the apparatus.

The results are given by the following system of partial differential equations:

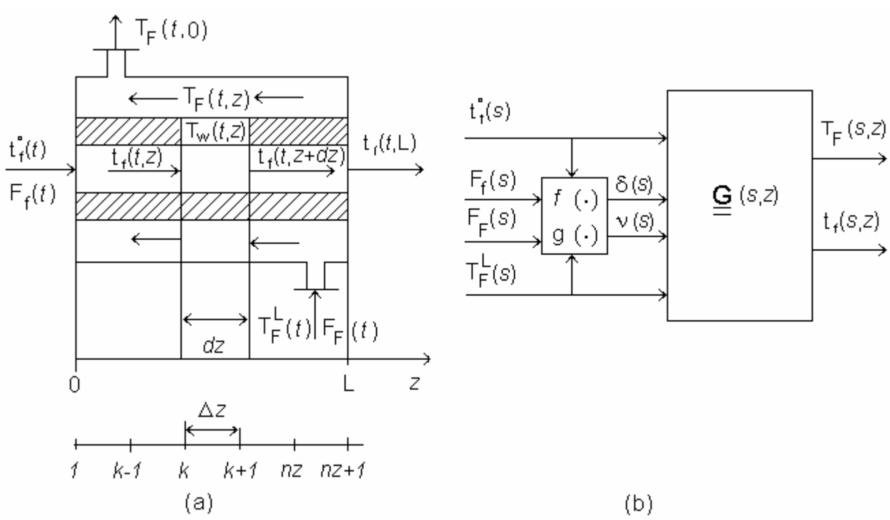

Fig. 1. Apparatus and time domain variables (a),Hammerstein model block diagram (b).

$$\frac{\partial t_f(t,z)}{\partial t} + v_f(t)\frac{\partial t_f(t,z)}{\partial z} = \gamma(t)\left[T_F(t,z) - t_f(t,z)\right] \qquad (1)$$

$$\frac{\partial T_F(t,z)}{\partial t} - V_F(t)\frac{\partial T_F(t,z)}{\partial z} = -\eta(t)\left[T_F(t,z) - t_f(t,z)\right] \qquad (2)$$

with split boundary conditions:

for $z = 0$ and $t > 0$ $\quad t_f(t,0) = t_f^0(t)$ (3)

for $z = L$ and $t > 0$ $\quad T_F(t,0) = T_F^L(t)$ (4)

and initial conditions:

for $0 \le z \le L$ and $t = 0$ $\quad t_f(0,z) = \bar{t}_f(z)$ (5)

" " $\quad T_F(0,z) = \bar{T}_F(z)$ (6)

where $\gamma(t)$ and $\eta(t)$ are quantities easily derivable.

The time dependence of parameters γ and η indicate that they can depend in any, even nonlinear, way from input variables $T_F^L(t)$, $t_f^0(t)$, $v_f(t)$, $V_F(t)$ and their combinations.

The above formulation leads to a semilinear system of equations. So the first step is to tackle it with a combination of characteristic and Laplace transform methods (Finlayson, 1992).

2.1 Characteristic-Laplace transform-Difference method

The characteristic method is particularly useful for converting hyperbolic partial differential equations into systems of ordinary differential equations. In case of semilinear problems with time dependent coefficients the characteristics are straight. Solutions can be found also in the Laplace domain (Rhee et al., 1986; Finlayson, 1992). In this study an explicit type marching solution is found. This method can be applied to the original system (1) - (6) as well as to that in deviation variables. With absolute formulations, Eqs. (1) - (6), the characteristic method gives:

$$\frac{dt_f(t,z)}{dt} = \gamma(t)\left[T_F(t,z) - t_f(t,z)\right]\Big|_{\alpha} \qquad (7)$$

$$\frac{dT_F(t,z)}{dt} = -\eta(t)\left[T_F(t,z) - t_f(t,z)\right]\Big|_{\beta} \qquad (8)$$

where characteristics α and β have the following equations on a *t, z* reference frame:

$t - z/v_f(t) = \text{const}$ (9)

$t + z/V_F(t) = \text{const}$ (10)

As assumed before regarding input types, parameters γ, and η are constant within the step, as well as $v_f(t)$ and $V_F(t)$. Should any of their variations be faster, appropriate sampling time must be chosen accordingly. So Laplace transforms of Eqs. (7) and (8), assuming non zero initial conditions and solved for the unknown:

$$t_f(s,z) = \left. \frac{t_f(0,z)}{s+\gamma} + \frac{\gamma T_F(s,z)}{s+\gamma} \right|_{\alpha} \tag{11}$$

$$T_F(s,z) = \left. \frac{T_{t_F}(0,z)}{s+\eta} + \frac{\eta t_f(s,z)}{s+\eta} \right|_{\beta} \tag{12}$$

Eqs. (11) and (12) are solutions for the state variables of the system reported in Fig. 1b. The output variables $t_f(\underline{t},\underline{z})$ and $T_F(\underline{t},\underline{z})$ are calculated afterwards in the following way. Denoting Δt the time increment, Δz the spatial mesh size, *i* the current time index and *k* the spatial index, and actually assuming that the forcing term is an arithmetic average between time i and (i-1) (Tan and Spinner, 1984), Eqs. (11) and (12) can be antitransformed (Ogunnaike and Ray, 1994) to:

$$t_f(i,k) = \phi_i\, t_f(i\text{-}1,k\text{-}1) + (1-\phi_i)[T_F(i,k)+T_F(i\text{-}1,k)]/2 \tag{13}$$

$$T_F(i,k) = \varphi_i\, T_F(i\text{-}1,k) + (1-\varphi_i)[t_f(i,k)+t_f(i\text{-}1,k)]/2 \tag{14}$$

where:

$$\phi_i = e^{-\gamma_i \Delta t} \tag{15}$$

$$\varphi_i = e^{-\eta_i \Delta t} \tag{16}$$

Solving Eqs. (13) and (14) for the unknown $t_f(i,k)$:

$$t_f(i,k) = \frac{\phi_i t_f(i\text{-}1,k\text{-}1) + \frac{(1-\phi_i)}{2}\left[\varphi_i T_F(i-1,k) + T_F(i-1,k-1) + \frac{(1-\varphi_i)}{2} t_f(i\text{-}1,k)\right]}{1 - \frac{(1-\phi_i)(1-\varphi_i)}{4}} \qquad \begin{matrix} i > 1 \\ 2 \le k \le nz \end{matrix} \tag{17}$$

is obtained. Eq. (17) contains quantities of one previous time step only, $t_f^0(i) = t_f^0(t)$, $v_f(i) = v_f(t)$ and $V_F(i) = V_F(t)$. Also time and input dependent parameters are calculated explicitly at each time step. The other unknown $T_F(i,k)$ can be calculated with an equation similar to Eq. (17) or alternatively with Eq. (14). However this marching solution is valid for internal points only. Different situations arise at the two boundaries. All of them can be handled with a procedure soon after described. The influence of inputs $v_f(t)$ and $V_F(t)$ is sensed through Eqs. (9) and (10) and , indirectly, through Eqs. (15) and (16).

However Δt and Δz cannot be independent. Fixed the spatial grid with mesh Δz, Δt must be equal to:

$$\Delta t = \Delta z / [v_f(t) + V_F(t)] \tag{18}$$

Referring to Fig. 2, the solutions at internal points other than those at the grid points can be calculated by defining a continuous parameter λ which is function of t, current time between t_i and t_{i-1}. Then Eqs. (13) and (14) applied two times allow to calculate explicitly the unknown $T_F^1(t,z)$ along the α characteristic as:

$$T_F^1(t,z) = \frac{\left(e^{-\gamma_i \lambda(t)\Delta t} - \phi_i\right) T_F(i,k) + \left(1 - e^{-\gamma_i \lambda(t)\Delta t}\right) T_F(i-1,k-1)}{1-\phi_i} \tag{19}$$

The other unknown $t_f^1(t,z)$ can be calculated as well:

$$t_f^1(t,z) = e^{-\gamma_i [1-\lambda(t)]\Delta t}\, t_f(i\text{-}1,k\text{-}1) + \{1- e^{-\gamma_i [1-\lambda(t)]\Delta t}\}[\, T_F^1(t,z) + T_F(i\text{-}1,k)]/2 \tag{20}$$

The same equations can be applied to point 7 and similar equations, omitted for brevity, can be derived for point 3. From points 1 and 3 point such as 2 can be calculated from the couple equations (13) and (14). Other points can be resolved, such as 4 and 6 and from them point 5; from point 5 and 7 point 8 can be known. Of course appropriate

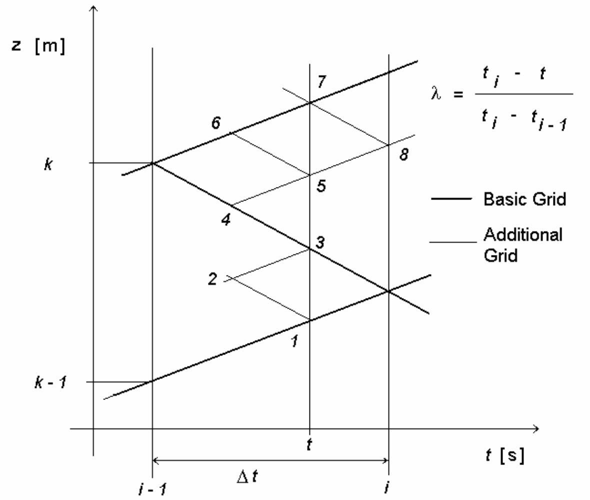

Fig. 2 . Refining Procedure

values of the parameter λ must be used. In this way continuity is recovered and *t* and *z* coordinate can be chosen at will covering the whole domain including end points.
Unfortunately natural boundary conditions at boundaries between patches, that is equal function value and first order derivatives in two neighbours patches, cannot be imposed. However the guarantee that a new steady state solution will be achieved would enforce the mentioned conditions to be satisfied to a great extent.

The transitories present discontinuities in 0^{th} derivatives during the first developing wave of both unknowns if $t_f^0(s)$ and $T_F^L(s)$ are step inputs respectively and in 1^{th} order derivatives of any of the two if only the other is step varying. The following waves present always discontinuities in 1^{th} order derivatives only. Only discontinuities in 1^{th} order derivatives are also present for step variations of the other two inputs $F_F(t)$ and $F_f(t)$ even in the first waves.

3. Results and Discussion

In this section I will report some results of preliminary calculations performed with the procedure developed in this work. The calculations obtained in the same apparatus and operating conditions by the Numerical Laplace Antitransform Method (NLAM), and by the Finite Element Method (FEM) are also reported for comparison purposes. The apparatus dimensions are reported in Table I, while steady state operating conditions are reported in Table II. The latter can be arbitrarily chosen without introducing any type of errors. For simplicity sake fluid properties have been kept constant for both cold and hot fluids and equal to that of water and the value of $R_d = 0$. The heat transfer coefficients, instead, are let to vary according to literature relationships, in this case the same as that reported in (Tan and Spinner, 1978), that is, dependence only on velocities through the exponent n_f equal to 0.8 for the cold fluid and n_F equal 0 for the hot fluid.
Some sample calculations have been performed varying all inputs at the same time. The variation of the inlet temperature of the cold fluid has been limited to **-** 10 °K while that of the hot fluid has been fixed to – 20 °K. Bigger variations of the fluid flow rates have been tested, that is $2.04{\cdot}10^{-4}$ m^3 / s for the cold fluid, which is almost four times that at steady state and that of the hot fluid has been set to $2.01{\cdot}10^{-4}$ m^3 / s which is nearly double that at steady state. Anyhow it should be pointed out that the solutions are valid for any variation of inputs and not restricted as in linearized models. However the results should be checked for fluid properties and heat transfer coefficients variations along the apparatus if higher temperature variations would be experienced. Fig. 3 shows threedimensional plots of the temperatures of the cold fluid obtained by the four calculation methods early mentioned. As can be seen the agreement between the four methods is satisfactory for engineering calculations, as outlined in e) and f) that represent a plot of the errors, differences between c) and a) surfaces and c) and d) surfaces respectively. Bigger discrepancies (localized spikes) are noticed in e) along disturbances paths because of step variations of inputs and because the prescribed (fixed) grid is not able to follow disturbances waves, unless finer grid meshes are used. Bigger discrepancies are noticed in the comparison with FEM because the latter introduce numerical dispersion, spurious oscillations and 1^{th} order derivatives mismatch.
Plots of the temperatures of the hot fluid are less interesting because successive waves vanish more rapidly and omitted for brevity.

Table I. Apparatus dimensions.

$D_i = 1.2 \cdot 10^{-2}$ m	$d_0 = 7.70{\cdot}10^{-2}$ m	$d_i = 6.56{\cdot}10^{-2}$ m	L = 20 m

Table II. Steady state conditions

$t_{fs}^0 = 288$ °K	$T_{Fs}^L = 325$ °K
$F_{fs} = 5.50{\cdot}10^{-5}$ m^3 / s	$F_{Fs} = 1.08{\cdot}10^{-4}$ m^3 / s
$h_{is} = 0.2$ Kcal / s m^2 °K	$h_{0s} = 0.1$ Kcal / s m^2 °K

4. Conclusions

A new method for simulating dynamics of double pipe heat exchangers has been developed and validated. It consists of explicit marching procedure for calculating transitory profiles of cold and hot fluid temperature generated when disturbances in temperature and flow rate of both fluids enter the equipment. It has been developed combining the main features of characteristic, Laplace transform and difference equation methods. Accepts stepwise variation of any combination of inputs. Non-zero generic initial conditions and any time dependence of parameters can be easily handled.

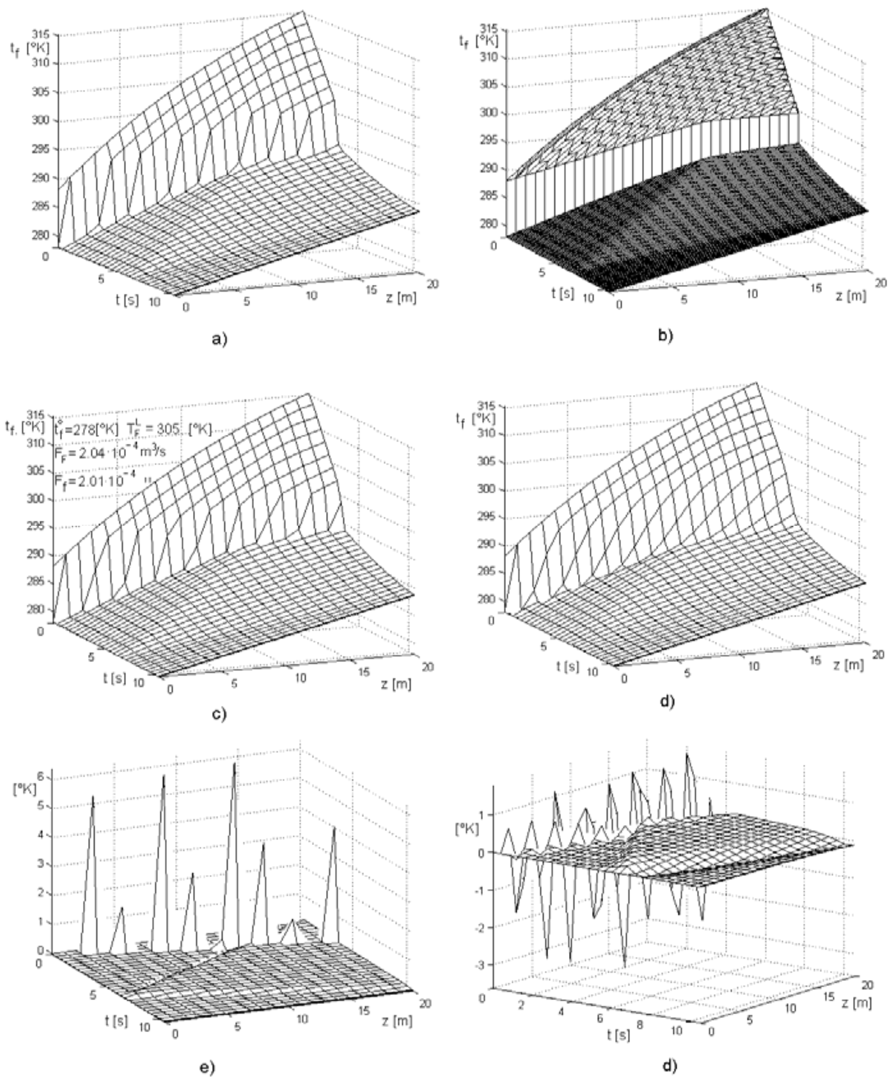

Fig. 3. Temperature profiles and errors of the cold fluid when the exchanger is operating with the new conditions reported in c): a) Numerical Laplace, b) Characteristics with natural coordinates, c) Characteristics with prescribed coordinates, d) Finite Element Method, e) c – a, f) c – d.

References

Ames, F. William (1977b) *Numerical Methods for Partial Differential Equations.* New York, Academic Press, Inc.. p. 329-332.

Evangelista (2005) Dynamics of Shell and Tube Heat Exchangers: New Insigths and Time Domain Solutions, *7th World Congress of Chemical Engineering*, Glasgow 10-14 July.

Finlayson, B. A. (1992) Numerical Methods for Problems with Moving Fronts. Seattle, W, Ravenna Park Publishing, Inc., p. 357.

Hollenbeck, K. J. (1998) INVLAP.M: A Matlab Function for Numerical Inversion of Laplace Transforms by the de Hoog Algorithm, http://www.isva.dtu.dk/staff/karl/invlap.htm.

Klinkenberg, A. (1954) Heat Transfer in Cross-Flow Heat Exchangers and Packed Beds. *Ind. Eng. Chem.*, 46(1), 2285-2289.

Koppel L. B. (1962) Dynamics of a Flow-Forced Heat Exchanger. *Ind. Eng. Chem. Fundam.*, 1(2), 131-134.

Lachi M., N. El Wakil and J. Padet (1997) The Time Constant of Double Pipe and One Pass Shell-and-Tube Heat Exchangers in the Case of Varying Fluid Flow Rates*. Int. J. Heat Mass Transfer* 40(9) 2067-2079.

Malinowsky L. and S. Bielsky (2004) An Analytical Method for Calculation of Transient Temperature Field in the Counter-Flow Heat Exchangers. *Int. Comm. Heat Mass Transfer* (31(5) 683-691.

Mozley, J. M. (1956) Predicting Dynamics of Concentric Pipe Heat Exchangers. *Ind. Eng. Chem.*, 48(6), 1035-1041.

Ogunnaike A. Babatunde and Ray W. Harmon (1994) *Process Dynamics, Modeling and Control.* Oxford, Oxford University Press. p. 1203-1206.

Ray, W. H. (1966) Forced-Flow Heat Exchanger Dynamics. *Ind. Eng. Chem. Fundam.*, 5(1), 138-139.

Rhee, H. K., R. Aris and N. Amundson (1986) First Order Partial Differential Equations, Vol. 1, Theory and Application of Single Equations. Englewood Cliffs, NJ, Prentice-Hall. p. 146.

Shampine L. F. and M. W. Reichelt (1997) The MATLAB ODE Suite. *SIAM J. Sci. Comp.*, 18(1), 1-22.

Steiner M.(1987) Dynamic Models for Heat Exchangers. *Proceedings of the XVIII Congress: The Use of Computers in Chemical Engineers.* Giardini Naxos (Italy) 26-30 April. pp. 809-814.

Tan, K. S. and I. H. Spinner (1978) Dynamics of a Shell-and-Tube Heat Exchanger with Finite Tube-Wall Heat Capacity and Finite Shell-Side Resistance. *Ind. Eng. Chem. Fundam.*, 17(4), 353-358.

Tan, K. S. and I. H. Spinner (1984) Numerical Methods of Solutions for Continuous Countercurrent Processes in the Nonsteady State, Part I: Model Equations and Development of Numerical Methods and Algorithms. *AIChE J.*, 30(5), 770-779.

Tan, K. S. and I. H. Spinner (1991) Approximate Solutions for Transient Response of a Shell and Tube Heat Exchanger. *Ind. Eng. Chem. Res.*, 30(7), 1639-1648.

Xuan, Y. and W. Roetzel (1993) Dynamics of Shell-and-Tube Heat Exchangers to Arbitrary Temperature and Step Flow Variations. *AIChE J.*, 39(3), 413-421.

Yin, J. and M. K. Jensen (2003) Analytic Model for Transient Heat Exchanger Response. *Int. J. Heat Mass Transfer*, 46(2003) 3255-3264.

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